Cantilever Type Stairs

Stairs

- Stairs are needed to transfer people vertically in the building between levels.

- Stair’s width 0.9 – 1.5m and stair well is 0.3 – 0.6m
Stairs’ Components

- **Landings**

- **Flights**: contains 10–14 steps; each has
  - **Risers**: 150 – 200mm
  - **Going (treads)**: 250 – 300mm

Structural System for Cantilever Stairs
Forces & Reinforcement

Minimum Dimensions

\[ t^* = \frac{t_s}{\cos \theta} \]

\[ t_{avg} = t^* + \frac{\text{riser}}{2} \]

\[ L_{eff} = \min \left\{ L_{clear} + t_{avg} \right\} \text{ edge to C.L.} \]
Loading

- **Dead Loads**
  - Self weight = $t_{avg} \times \gamma_{conc}$
  - Covering = $0.8 \rightarrow 1.0$ kN/m$^2$

- **Live Loads**
  - $3 \rightarrow 4$ kN/m$^2$

- **Parapet Load**
  - Concentrated L.L. @ edge = 1.5 kN/m'

Example
Example

Design a cantilever type stair case for a floor height of 3.0m

**Loads:**
- Covering material 0.8kN/m²
- Live Load 3.0 kN/m²

**Materials:**
- $f_{cu} = 35N/mm²$
- $f_y = 400 N/mm²$

Solution

1] Design of the Flight

Assume the riser height is 150mm and the going width is 300mm.

$$\theta = \tan^{-1}(\text{riser/going}) = \tan^{-1}(150/300) = 26.56°$$

1.1) Load Calculations

Assume $t_s = 140mm$

$$t' = \frac{t_s}{\cos \theta} = \frac{140}{\cos(26.56)} = 156.5mm$$

$$t_{avg} = t' + \frac{\text{riser}}{2} = 156.5 + \frac{150}{2} = 231.5mm$$
Solution

1.1) Load Calculations (cont’d)
   Dead Load = $\gamma_{\text{conc}} t_{\text{avg}} + \text{covering materials}
   = 25 \times 0.2315 + 0.8 = 6.5875 \text{ kN/m}^2

   \[ W_u = 1.4DL + 1.6LL = 1.4 \times 6.5875 + 1.6 \times 3 \]
   = 14.023 kN/m^2

   \[ P_u = 1.6 \times 1.5 = 2.4 \text{ kN/m}’ \]

1.2) Effective Length

   \[ L_{\text{effective}} = \min \left\{ \frac{L_{\text{clear}} + t_{\text{avg}}}{\text{edge to CL}} \right\} = \min \left\{ \begin{array}{l}
       1.6 + 0.2315 = 1.8315m \\
       1.6 + 0.125 = 1.725m
     \end{array} \right\} \]

1.3) Bending Moments

   \[ M_u = W_u l^2/2 + P_u l \\
   = 14.02 \times 1.725^2/2 + 2.4 \times 1.725 = 25 \text{ kN.m/m}’ \]

1.4) Design of Reinforcement

   Assume 20mm cover → $d = t_{\text{avg}} - 20\text{mm} = 211.5\text{mm}$

   \[ R_i = \frac{M_u}{f_{\text{cd}} b d^2} = \frac{25 \times 10^6}{35 \times 1000 \times 211.5^2} = 0.016 \]

   from curves $\omega = 0.019$

   \[ A_i = \omega \frac{f_{\text{cd}}}{f_{\text{y}}} b d = 0.019 \times \frac{35}{400} \times 1000 \times 211.5 = 346 \text{mm}^2 \]
Solution

1.5) Check for Minimum Reinforcement

\[ A_{r,\text{min}} = \frac{0.6}{f_y} \cdot bd = \frac{0.6}{400} \cdot 1000 \times 211.5 = 317 \text{mm}^2 < A_r \quad \text{\(\checkmark\ ok\)}\]

\[ A_{r}/\text{step} = \text{As x step width (going)} \]
\[ = 347 \times 0.30 = 104 \text{mm}^2 \]

choose 2 \(\phi\) 10 (157\text{mm}^2)/step

Solution

2] Design of The Landing

take thickness \(= t_s = 140\text{mm}\)

\[ W_u = 1.4DL + 1.6LL \]
\[ = 1.4(25 \times 0.14 + 0.8) + 1.6 \times 3 = 10.82 \text{kN/m}^2 \]

As an approximation take half of the load in each direction; \(W_{ul} = 10.82/2 = 5.41 \text{kN/m}'\)

\[ M_u = W_{ul} \times L^2 \text{ effective}/2 = 5.41 \times 1.725^2/2 = 8.05 \text{kN.m} \]

Take \(d = t_s - 20\text{mm} = 120\text{mm}\)
Solution

2] Design of The Landing (cont’d)

\[ R_i = \frac{M_i}{f_{\omega} \cdot b d^2} = \frac{8.05 \times 10^6}{35 \times 1000 \times 120^2} = 0.0159 \]

From curves → \( \omega = 0.019 \)

\[ A_i = \omega \frac{f_{\omega}}{f_{\nu}} \cdot b d = 0.019 \cdot \frac{35}{400} \times 1000 \times 120 = 196 \text{mm}^2 \]

Choose 5φ10mm (393mm\(^2\)) > \( A_{s,\text{min}} \), √ ok

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Solution

3] Design of the Supporting Beam

3.1) Calculation of Loads

Assume the beam cross section is 250 x 800mm.

To obtain weight in the hαl projection use \( t^*_{b} \)

\[ t^*_{b} = \frac{t}{\cos \theta} = \frac{800}{\cos(26.56)} = 894.4 \text{mm} \]

Self weight = 1.4*\( y_c \cdot b \cdot t^*_{b} = 1.4 \times 25 \times 0.25 \times 0.8944 \]

= 7.83 kN/m’

\( h_w\) = 3-floor beam-\( t^*_{b} \cdot 1.5/2 = 3 - 0.6 - 0.89 - 0.75 \)

= 0.755m
Solution

3.1) Calculation of Loads (cont’d)

Wall load = \(1.4(\gamma_w t_w + 0.9)h_w\)
\[= 1.4(16 \times 0.25 + 0.9) \times 0.755 = 5.18 \text{kN/m}’\]

\(W_u\) = self wt. + \(W_u\)xflight width + wall load + edge live load
\(W_{ubf} = 7.83 + 14.02 \times 1.6 + 5.18 + 2.4 = 37.84 \text{kN/m}’\)

The load transmitted to the beam @ the landing level
\(W_{UBL} = \) self wt. + \(W_u\)xlanding width + wall load
\(W_{UBL} = 7.83 + 5.41 \times 1.6 + 5.18 = 21.67 \text{kN/m}’\)
Solution

3.2) Design for Flexure

\[ R_1 = \frac{37.84 \times 3 \times 1.6 + 1.5}{4.6} + 21.67 \times 1.6 \times 0.8 = 82.53 \text{kN} \]

\[ R_2 = \frac{37.84 \times 3 \times 1.5 + 21.67 \times 1.6 \times 3 + 0.8}{4.6} = 65.66 \text{kN} \]

Location of maximum moment \( x' \)

\[ x' = \frac{R_1}{W_{abf}} = \frac{82.53}{37.84} = 2.18 \text{m} \]

\[ M_{\text{max}} = R_1 x' - W_{abf} \frac{x'^2}{2} \]

\[ = 82.53 \times 2.18 - 37.84 \times \frac{2.18^2}{2} = 90 \text{ kN.m} \]

Solution

3.2) Design for Flexure (cont’d)

\[ R_i = \frac{M_i}{f_{ca} bd^2} = \frac{90 \times 10^6}{35 \times 250 \times 750^2} = 0.0183 \]

From Curve \( \rightarrow \omega = 0.02196 \)

\[ A_s = \omega \frac{f_{ca} bd}{f_y} = 0.02196 \times \frac{35}{400} = 0.250 \times 750 = 360.3 \text{mm}^2 \]

Check for minimum reinforcement

\[ A_{s,\text{min}} = \text{smaller of} \left\{ \begin{array}{l}
0.225 \frac{f_{ca} bd}{f_y} = 0.225 \times \frac{35}{400} = 250 \times 750 = 623 \text{mm}^2 < A_s \\
1.3A_s = 1.3 \times 360.3 = 468.4 \text{mm}^2
\end{array} \right. \]

\[ A_s < A_{s,\text{min}} \rightarrow \text{use } A_{s,\text{min}} = 468.4 \text{mm}^2 \] (Choose \( 3\phi 16, 603 \text{mm}^2 \))
Solution

3.3) Design for Shear & Torsion

due to inclination the reaction or any vertical force on the beam will result in:

shear; \( Q = V \cos \theta \)
Normal; \( N = V \sin \theta \)

3.3.1) Shear Stresses

The critical section is @ d/2 from face of support;
Assuming the column width is 600mm → critical section is at distance = (0.6/2 + 0.75/2)

\[ V = R_1 - W_{ubf} (0.6/2 + 0.75/2) = 82.53 - 37.84 (0.3 + 0.375) = 57 \text{ kN} \]

\[ Q_u = V \cos \theta = 57 \times \cos(26.56) = 51 \text{ kN} \]

3.3.1) Shear Stresses (cont’d)

\[ q_u = Q_u / bd = 51 \times 10^3 / (250 \times 750) = 0.272 \text{ N/mm}^2 \]

3.3.2) Shear Reinforcement

\[ q_{cu} = 0.24 \sqrt{f_{cu}} \quad \& \quad q_{umax} = 0.7 \sqrt{f_{cu}} \]

\[ q_{cu} = 0.24 \sqrt{35 / 1.5} = 1.16 \text{ N/mm}^2 \quad \& \quad q_{umax} = 0.7 \sqrt{35 / 1.5} = 3.38 \text{ N/mm}^2 \]

\[ q_u < q_{cu} \rightarrow \text{shear reinforcement is not needed} \]
Solution

3.3.3) Torsion Stresses

\[ x_1 = 250 - 2 \times 40 = 170 \text{mm} \]
\[ y_1 = 800 - 2 \times 40 = 720 \text{mm} \]
\[ p_h = 2x(x_1 + y_1) = 2x(170 + 720) = 1780 \text{mm} \]
\[ A_{oh} = x_1 \cdot y_1 = 170 \times 720 = 122400 \text{mm}^2 \]
\[ A_0 = 0.85A_{oh} = 0.85 \times 122400 = 104040 \text{mm}^2 \]
\[ t_e = A_{oh}/p_h = 122400/1780 = 68.76 \text{mm} \]

Critical section for torsion is @ d/2 from face of support

\[ T_u = 52.8 - 25 (0.6/2 + 0.75/2) \]
\[ = 35.9 \text{ kN.m} \]

\[ M_{Tu} = T_u \cdot \cos \theta = 35.9 \times \cos 26.56^\circ \]
\[ = 32.1 \text{ kN.m} \]
\[ \rightarrow q_{tu} = \frac{M_{Tu}}{2A_0 t_e} = \frac{32.1 \times 10^6}{2 \times 104040 \times 68.76} = 2.24 \text{ N/mm}^2 \]
Solution

3.3.4) Check for adequacy of concrete dimensions

\[
\sqrt{q_n^2 + q_m^2} \leq q_{u,\text{max}}
\]

\[
\sqrt{0.272^2 + 2.24^2} = 2.26 \leq 3.38 \quad \checkmark \text{ok}
\]

3.3.5) Torsional Reinforcement

\[
q_{u,\text{min}} = 0.06 \sqrt{\frac{f_{cu}}{\gamma_c}} = 0.06 \sqrt{\frac{35}{1.5}} = 0.29 N/mm^2
\]

\(Q_{tu} > q_{tu,\text{min}} \rightarrow \text{torsional reinforcement is needed}\)

Assume stirrups spacing is 100mm

\[
A_{st} = \frac{M_{u}S}{2A_{y} \left( \frac{f_{y,\text{st}}}{\gamma_t} \right)} = \frac{32.1 \times 10^6 \times 100}{2 \times 104040 \times \left( \frac{240}{1.15} \right)} = 73.97 mm^2
\]

Solution

3.3.5) Torsional Reinforcement (cont’d)

\[
A_{sl} = \frac{A_{str} P_h}{S} \left( \frac{f_{y,\text{st}}}{f_y} \right) = \frac{73.97 \times 1780}{100} \left( \frac{240}{400} \right) = 790 mm^2
\]

Check for minimum longitudinal reinforcement

\[
A_{sl,\text{min}} = \frac{0.4 \sqrt{\frac{f_{cu}}{\gamma_c}} A_{y}}{f_y / \gamma_t} - \left( \frac{A_{str}}{S} \right) P_h \left( \frac{f_{y,\text{st}}}{f_y} \right)
\]

\[
A_{sl,\text{min}} = \frac{0.4 \sqrt{\frac{35}{1.5}} 250 \times 800}{400/1.15} - \left( \frac{73.97}{100} \right) 1780 \left( \frac{240}{400} \right) = 320 mm^2 \quad \checkmark \text{ok}
\]

Choose 8\(\phi\)12 (904mm\(^2\))
Solution

3.3.6) Reinforcement for combined shear and torsion

\[ A_{str} + A_{st}/2 = 73.97 + 0 = 73.97 \text{mm}^2 \]

Choose \( \phi 10 \text{mm} @ 100 \text{mm} (78.5 \text{mm}^2) \)

Check for minimum stirrups reinforcement

\[ A_{st,min} = (0.4/f_y)b.s = (0.4/240)250 \times 100 = 41.67 \text{mm}^2 \]

Total area of stirrups = \( 2A_{str} + A_{st} = 2 \times 73.97 + 0 = 147.94 \text{mm}^2 \)

\[ > A_{st,min} \quad \checkmark \text{ok} \]
Solution