



Alexandria University Faculty of Science Department of Mathematics and Computer Science

Solutions of Some Flow Problems of Newtonian Fluids through Porous Media

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Science

In

Mathematics (Applied Mathematics)

Presented by

Ahmed Saeed Ibrahim Amer

B.Sc. Science (Mathematics-Computer science),

Faculty of Science, Alexandria University, 2015

2020

Department of Mathematics - Faculty of Science - Alexandria University

Summary

The thesis is concerned with the motion of viscous fluid in porous media using Brinkman model and analysis of some new conditions is added to it, one of them an approximate sphere translate in porous medium, and the other is rotate it in the porous media, then we calculate the drag force and torque for it. This thesis also includes an analytical investigation for the thermocapillary and photocapillary of spherical droplet embedded in porous medium. The results are represented graphically for various physical parameters for all problems.

Chapter one, in this chapter some basic proprieties of polyadic algebra are introduced. Furthermore, a discussion of Navier-Stokes's equation in clear and porous media is given. Definitions of some physical concepts such as porosity, Permeability and viscosity are given. Fundamentals of heat transfer through porous media and theory of photophoresis are presented.

In Chapter two, in this chapter, we consider the axisymmetric flow of viscous fluid through porous medium. A spheroidal particle embedded in this medium. The following assumptions are made: the porous medium is modeled by the Darcy-Brinkman equation and the flow is considered to be slow. An analytical solution is obtained for the stream function. The drag acting on the spheroidal particle is calculated up to $O(\epsilon^2)$, where ϵ is the deformation parameter characterizing the shape of the particle. The results for the drag force coefficient versus the permeability parameter, and deformity parameter are presented in graphical forms. Also, the results for the drag coefficient are compared with the available solutions in the literature for the limiting cases of Stokes clear fluid and Darcy's flow. Also, the velocity field about a spheroidal particle rotating in a Brinkman fluid is derived and the hydrodynamic torque is also calculated up to $O(\epsilon^2)$. The effect of the Darcy permeability on the torque is compared with the translational drag. The Brinkman effect on rotation is substantially weaker than on translational motion for any value of the permeability parameter.

In Chapter three, the velocity field around a spheroidal solid particle rotating about z- direction in a Brinkman fluid is derived and the hydrodynamic torque is calculated. The viscous flow through the porous medium is considered to be slow and axisymmetric. The torque acting on the spheroidal particle is calculated up to $O(\epsilon^2)$, where ϵ is the deformation parameter characterizing the shape of the particle. The effect of the Darcy permeability on the torque is compared with the translational drag. The Brinkman effect on rotation is substantially weaker than on translational motion for any value of the permeability parameter.

In Chapter four, an analytical study is presented for thermocapillary migration of a spherical fluid droplet embedded in a porous medium. The porous medium is assumed to be homogeneous, isotropic and the solid matrix is in thermal equilibrium with the fluid through the voids of the medium. A constant prescribed temperature gradient parallel to an adiabatic plane is maintained in the porous medium. The Peclet number is assumed to be small, so that the temperature distribution is governed by Laplace equation. Also, Reynolds number is assumed to be small so that the fluid flow inside the droplet is described by Stokes equation and the flow inside the porous medium is described by Brinkman equation. Expressions for thermocapillary and photocapillary velocities and forces are obtained.

i List of Figures

The effect of the Brinkman number characterizing the permeability of the medium is investigated as functions of the thermal properties of the porous medium and droplet. The limiting cases of Stokes and Darcy's flows are discussed.